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An investigation on the interaction analysis of beam- active isolator with low suspension frequency

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ABSTRACT

The application of vibration isolation with particular low or high supporting stiffness is widely used in the field of science and engineering. These particular supporting stiffness can be provided by using active vibration isolation. The supported structure is connected to the active isolation unit, therefore creating an interaction between the structure vibration behaviour and the isolation unit dynamic characteristics. This work investigates the interaction between a structure and an active isolator for a low stiffness support to design an accurate practical isolation system. It is found that the structure provides additional mass, stiffness and force to the active isolator. This shows that the interaction affects the active isolator and this must be considered when designing a practical isolation unit.

INTRODUCTION

Currently in the field of science and engineering there is a need for vibration isolation systems with particular low or high suspension frequency. Large civil aircrafts that undergo ground vibration tests (GVT) require a suspension frequency of less than 1/3 of the system's fundamental frequency [1]. GVT aims to provide modes of vibration of a free-free beam to an aircraft. It is therefore necessary to further develop methods of support for large thin- wing aircraft with fundamental frequencies less than 1Hz. To realize this condition, a method that has been found to provide suspension frequencies of less than 0.25 Hz for a limited range of movement, is by using nonlinear spring system [1]. Applications of GVT around the world include amongst others, at the National Aeronautics and Space Administration (NASA) Dryden Flight Research Center (Kehoe and Freudinger (NASA Technical Memorandum, 1994)) at the European Aerospace and Defence System (EADS) in Sogerma-Bordeaux, France and the Embraer in Sao Paolo, Brazil [2].

Apart from using the method of nonlinear spring system, another method that can provide particular vibration isolation performance requirements is by using active vibration isolation system. An active vibration isolation system provides active feedback control that can modify the dynamic stiffness of a passive system [3].

As discussed by Xing et al. [4] for the analysis of structure-control interactions, the dynamic characteristics of both structures and control system affect each other. Therefore, to assess the efficiency of an active isolation system, it is necessary to consider interactions. In a previous paper [5], some work has been done without considering interactions. The work is extended in this paper which aims to develop an integrated interaction analysis of a generalised active low suspension frequency system and its supported structure.

MATHEMATICAL MODEL OF AN INTEGRATED ACTIVE ISOLATOR-STRUCTURE INTERACTION

The active isolation unit was designed according to the work of Xing et al. [3]. Here, they illustrated several design strategies that can produce zero or infinite-dynamic modulus. A beam is attached on top of the active isolator to analyse the beam active-isolator interaction.

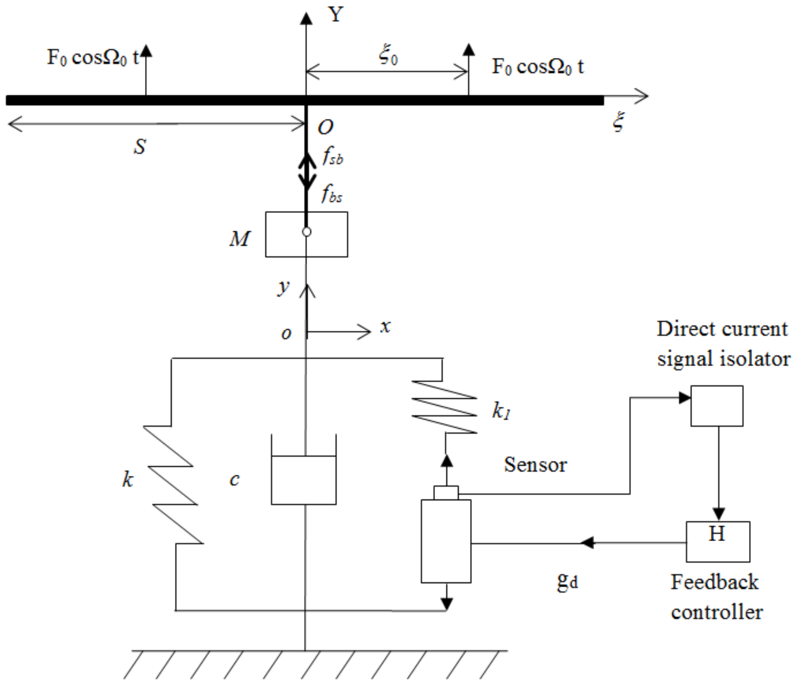


Figure 1: An integrated system consisting of an elastic beam and an active isolator unit.

Figure 1 shows an integrated interaction system in which an elastic structure is supported by an active vibration isolation unit. To simplify the mathematical analysis and maintain the essential characteristics of the problem, the structure is considered as a uniform elastic free-free beam subject to two harmonic forces $F_0 \cos\Omega_0 t$ applied symmetrically at point ξ_0 under the beam coordinate system $O-\xi Y$ fixed at the middle point O of the beam. There is a lumped mass M connected at point O by a rigid rod with its mass included in M. The beam is of span length $2S$, mass density ρ per unit length and bending stiffness $\Psi = EI$. Since the beam is elastic, its deflection $Y(\xi,t)$ is a function of beam material point ξ and time t. The lumped mass M is supported by an active isolation system and it moves in the y direction only. The active isolation system consists of k and c which denote the stiffness and damping coefficients of the mechanical device supporting the static weight of the aircraft wing. A spring of small stiffness k_1 is connected in series with an active displacement feedback controller. The feedback control system provides a dynamic force $f_c = g_d y$ proportional to the displacement. A direct current signal isolator is used so that the feedback system produces only alternating forces acting on the mass. Therefore, the feedback system does not affect the static stiffness of the supporting system.

Dynamic equilibrium equation and boundary conditions of beam structure

The dynamic equilibrium equation and boundary conditions of the beam structure are shown as below:

$$\Psi Y^{(\mathrm{IV})}+\rho\ddot{Y}=\delta(\xi-\xi_0)F_0\cos\Omega_0 t;\quad Y''=0=Y''',\quad \xi=S;Y'=0,\quad \Psi Y'''=f_{bs},\xi=0 \tag{1}$$

Here, $()'=\partial()/\partial\xi,\ \dot{()}=\partial()/\partial t,$ etc., and f_{bs} represents a dynamic shearing force acted on the beam section $\xi=0$ by the rigid rod, $\delta()$ denotes Delta function.

The deflection $Y(\xi,t)$ which represents the dynamic deflection of the beam relative to the static state of the beam is represented by a mode summation form:

$$Y(\xi,t)=\mathbf{Y}(\xi)\mathbf{\Phi}(t),\quad \mathbf{Y}=\begin{bmatrix}Y_1 & Y_2 & \cdots & Y_N\end{bmatrix},\quad \mathbf{\Phi}=\begin{bmatrix}\phi_1 & \phi_2 & \cdots & \phi_N\end{bmatrix}^T, \tag{2}$$

$$Y_n(\xi)=\frac{1}{2}\{\frac{\cosh(\lambda_n\xi/S)}{\cosh\lambda_n}+\frac{\cos(\lambda_n\xi/S)}{\cos\lambda_n}\},\qquad \tan\lambda_n+\tanh\lambda_n=0,\qquad n=1,2,3,\cdots.$$

based on the non-dimensional symmetrical mode functions $Y_n(\xi)$, $(n=1,2,...,N)$, of the uniform free-free beam. Here, N denotes a number of the retained mode functions $Y_n(\xi)$ and ϕ_n represents a generalised coordinate corresponding to mode n, which has a length dimension. These mode functions satisfy the following orthogonal relationships,

$$\int_0^S \mathrm{Y}_n''\mathrm{EIY}_m''\mathrm{d}\xi=\begin{cases}0, & \mathrm{n}\neq\mathrm{m},\\ \mathrm{K}_{\mathrm{nn}}, & \mathrm{n}=\mathrm{m},\end{cases}\qquad \int_0^S \mathrm{Y}_n\rho\mathrm{Y}_m\mathrm{d}\xi=\begin{cases}0, & \mathrm{n}\neq\mathrm{m},\\ \mathrm{M}_{\mathrm{nn}}, & \mathrm{n}=\mathrm{m},\end{cases}$$
$$\mathrm{M}_{\mathrm{nn}}=\begin{cases}\rho\mathrm{S}, & \mathrm{n}=1,\\ \rho\mathrm{S}/4 & \mathrm{n}\neq1,\end{cases}\quad \mathrm{K}_{\mathrm{nn}}=\begin{cases}0, & \mathrm{n}=1,\\ \dfrac{\lambda_n^4\Psi}{4\mathrm{S}^3} & \mathrm{n}\neq1,\end{cases}\quad \hat{\Omega}_{\mathrm{n}}=\sqrt{\mathrm{K}_{\mathrm{nn}}/\mathrm{M}_{\mathrm{nn}}}=\frac{\lambda_n^2}{\mathrm{S}^2}\sqrt{\frac{\Psi}{\rho}}. \tag{3}$$

The sub-index n indicates the mode number of the free-free beam, $\hat{\Omega}_n$, K_n and M_n represent the n-th natural frequency, generalised stiffness and mass, respectively. For the free-free beam, its first mode is a rigid mode with frequency $\hat{\Omega}_1=0$ and mode function $Y_1=1$.

Substituting Equation (2) into Equation (1) and using the orthogonal relationships (3), we obtain the following mode equation describing the beam motion

$$\mathbf{m}\ddot{\mathbf{\Phi}}+\mathbf{k\Phi}=\mathbf{Y}^{\mathrm{T}}(0)\mathrm{f}_{\mathrm{bs}}+\mathbf{Y}^{\mathrm{T}}(\xi_0)\mathrm{F}_0\cos\Omega_0\mathrm{t},$$
$$\mathbf{m}=\mathbf{diag}(\mathrm{M}_{\mathrm{nn}}),\qquad \mathbf{k}=\mathbf{diag}(\mathrm{K}_{\mathrm{nn}}),\qquad \mathbf{\Lambda}^2=\mathbf{diag}(\hat{\Omega}_{\mathrm{n}}^2). \tag{4}$$

Dynamic equilibrium equation of the active suspension system

The dynamic equilibrium equation of the active suspension system in Figure 1 is described by

$$M\ddot{y} + c\dot{y} + \left(k + \frac{k_1 g_d}{k_1 + g_d}\right) y = f_{sb} \qquad (5)$$

Here, the force f_{sb} denotes the reaction force from the beam to the lumped mass M .

Interaction conditions between the beam structure and the active suspension unit

On the interaction section $\xi = 0$ between the beam and the active suspension unit, a dynamic equilibrium condition and a geometrical constraint condition are required, i.e.

Equilibrium: $f_{bs} + f_{sb} = 0, \qquad -f_{bs} = f_{sb} = f,$ (6)

Geometrical constraint: $Y(0,t) = y(t),$ (7)

which, when Equation (2) used, is written in the mode form

$$\mathbf{Y}_0 \mathbf{\Phi} = \mathrm{y}, \qquad \mathbf{Y}_0 = \mathbf{Y}(0). \qquad (8)$$

INTERACTION ANALYSIS OF BEAM-ACTIVE ISOLATOR WITH LOW SUSPENSION FREQUENCY

From Equation (4) we have

$$\mathbf{m}\ddot{\mathbf{\Phi}} + \mathrm{m}\Lambda^2 \mathbf{\Phi} = \mathbf{Y}_0^T f_{bs} + \mathbf{Y}_F^T F_0 \cos\Omega_0 t, \qquad \mathrm{Y}_F = \mathrm{Y}(\xi_0) \qquad (9)$$

$$\ddot{\mathbf{\Phi}} + \Lambda^2 \mathbf{\Phi} = \mathrm{m}^{-1} \mathbf{Y}_0^T f_{bs} + \mathrm{m}^{-1} \mathbf{Y}_F^T F_0 \cos\Omega_0 t, \qquad (10)$$

Multiplying Equation (10) by $\mathbf{Y}_0$, leads to

$$\begin{aligned} \mathbf{Y}_0 \ddot{\mathbf{\Phi}} + \mathbf{Y}_0 \mathbf{\Lambda}^2 \mathbf{\Phi} &= \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_0^{\mathrm{T}} f_{bs} + \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_{\mathrm{F}}^{\mathrm{T}} \mathrm{F}_0 \cos\Omega_0 \mathrm{t} \\ &= m_b^{-1} f_{bs} + \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_{\mathrm{F}}^{\mathrm{T}} \mathrm{F}_0 \cos\Omega_0 \mathrm{t}, \qquad m_b^{-1} = \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_0^{\mathrm{T}} \end{aligned} \qquad (11)$$

Since

$$m_b^{-1} = \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_0^{\mathrm{T}} = \sum_{\mathrm{n}=1}^{\mathrm{N}} M_{nn}^{-1} \mathrm{Y}_{\mathrm{n}0}^2 > 0, \qquad (12)$$

$$m_b = \frac{1}{\sum_{n=1}^{N} \frac{Y_{n0}^2}{M_{nn}}} \qquad (13)$$

We have

$$f_{bs} = m_b \mathbf{Y}_0 \ddot{\mathbf{\Phi}} + m_b \mathbf{Y}_0 \overline{\mathbf{\Lambda}}^2 \mathbf{\Phi} - m_b \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_{\mathrm{F}}^{\mathrm{T}} \mathrm{F}_0 \cos\Omega_0 \mathrm{t} = m_b \ddot{\mathrm{Y}} + k_b (\mathbf{\Phi}) \mathrm{Y} - f_b \mathrm{F}_0 \cos\Omega_0 \mathrm{t} \qquad (14)$$

where

$$k_b(\mathbf{\Phi}) = m_b \frac{\mathbf{Y}_0 \mathbf{\Lambda}^2 \mathbf{\Phi}}{\mathbf{Y}_0 \mathbf{\Phi}} = \frac{m_b \sum_{n=1}^{N} Y_{n0} \hat{\Omega}_n^2 \phi_n}{\sum_{n=1}^{N} Y_{n0} \phi_n}, \qquad f_b = m_b \mathbf{Y}_0 \mathbf{m}^{-1} \mathbf{Y}_F^T. \tag{15}$$

Using Equation (6) we have

$$f_{sb} = -m_b \ddot{y} - k_b(\mathbf{\Phi}) y + f_b F_0 \cos \Omega_0 t \tag{16}$$

Substituting Equation (16) into Equation (5), we obtain

$$M\ddot{y} + c\dot{y} + \left(k + \frac{k_1 g_d}{k_1 + g_d} \right) y = -m_b \ddot{y} - k_b(\mathbf{\Phi}) y + f_b F_0 \cos \Omega_0 t$$

$$(M + m_b)\ddot{y} + c\dot{y} + \left(k + \frac{k_1 g_d}{k_1 + g_d} + k_b(\mathbf{\Phi}) \right) y = f_b F_0 \cos \Omega_0 t \tag{17}$$

Here, m_b and k_b represent an additional dynamic mass and stiffness, respectively, which are added to the active suspension system by the beam due to their dynamic interactions. f_b defines a force factor at which the excitation force is added to the lumped mass. The values of these added parameters depend on the retained mode number of the beam. The added stiffness k_b also involves the dynamic response $\mathbf{\Phi}$ of the beam. For a unit dynamic response of mode n, i.e. $\mathbf{\Phi}^T = [0 \ \cdots \ 0 \ \phi_n \ 0 \ \cdots \ 0]^T$, the added mass and stiffness are respectively obtained by Equations (18-1) and (18-2),

$$m_b(\phi_n) = M_{nn} Y_{n0}^{-2}, \tag{18-1}$$

$$k_b(\phi_n) = m_b \hat{\Omega}_n^2 = M_{nn} Y_{n0}^{-2} K_{nn} / M_{nn} = Y_{n0}^{-2} K_{nn}. \tag{18-2}$$

Here, we consider $N = 1$ implying that only one rigid mode $Y_1 = 1$ with $\hat{\Omega}_1^2 = 0$ in Equation (1) is retained, so that $\mathbf{Y}_0 = 1$, and therefore we have

$$\mathbf{m} = M_{11} = \rho S, \qquad m_b = M_{11}, \qquad k_b = 0, \qquad f_b = 1. \tag{19}$$

Physically, m_b in Equation (19) is the total mass of the beam. Since the beam is considered rigid and there is no elastic deformation, the added stiffness $k_b = 0$, and the force factor $f_b = 1$.

Table 1 and 2 show the values of the added parameters affected by retained mode number, N and values of the added parameters affected by a unit dynamic response mode, n.

Table 1: The values of added parameters affected by retained mode number N of the beam on the nonlinear suspension unit, $(\xi_0 = 1)$.

N	1	2	3	4	5
m_b	100	167.6685	217.0955	267.1206	317.1223
f_b	1.000	-0.9685	0.6986	-0.7075	0.7071

Table 2: The values of added parameters affected by a unit dynamic response mode n of the beam on the nonlinear suspension unit.

n	1	2	3	4	5
$m_b(\phi_n)$	100	67.67	49.43	50.03	50.00
$k_b(\phi_n)$	0	68	1443	8908	30787

From Table 1 it can be seen that the additional mass, m_b increases as the number of retained mode, N increases. The value of force factor is seen to be positive at odd numbers of retained modes and negative at even numbers of retained modes. The positive value of force factor defines a pulling force and a negative value implies a compressed force.

From Table 2 it can be seen that the added mass, $m_b(\phi_n)$ decreases as the mode number increases. However, the value of additional mass for mode numbers n = 3, 4, and 5 continue to have a similar value of approximately 50. The added stiffness, $k_b(\phi_n)$ for a unit dynamic response is seen to increase as the mode number, n increases.

CONTROLLER DESIGN FOR BEAM-ACTIVE ISOLATOR WITH LOW SUSPENSION FREQUENCY

In this section, a suitable negative feedback gain for the beam-active isolator that can provide a low suspension frequency was obtained and inserted into the active system to observe its response.

Finding suitable negative feedback gains that provide a low suspension frequency

From Equation (17), the frequency of the supporting system for a unit dynamic response mode n is given by

$$\Omega_n = \sqrt{\frac{k + \dfrac{k_1 g_d}{k_1 + g_d} + k_b(\phi_n)}{M + m_b(\phi_n)}} \qquad (20)$$

A suitably chosen negative feedback gain, g_d satisfying the condition

$$-\left(\frac{k_1(\mathrm{k}+k_b(\phi_n)}{k_1+k+k_b(\phi_n)}\right) < g_d < 0 \tag{21}$$

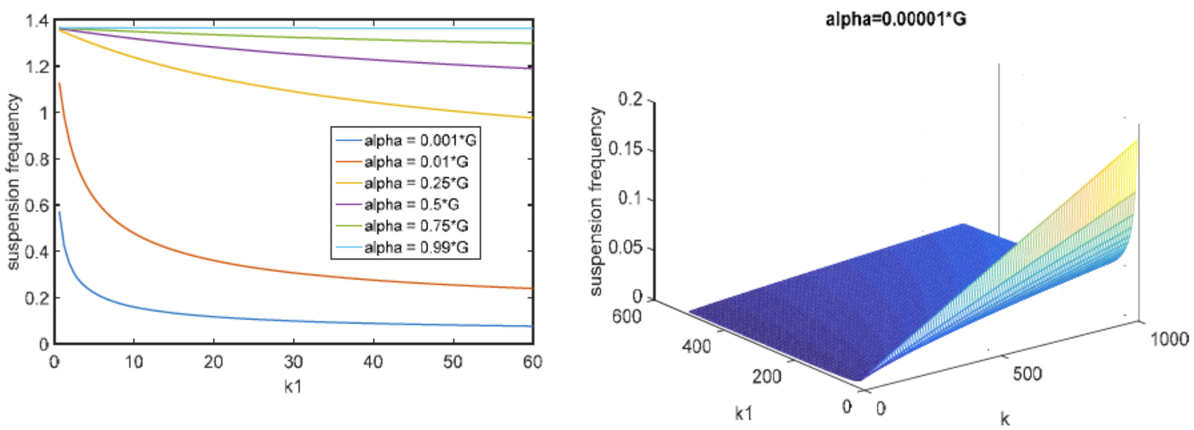
allows a reduced frequency to be achieved as indicated by Equation (20)

From Equation (21), introducing α, where

$$g_d = -\left(\frac{k_1(k+k_b(\phi_n)}{k_1+k+k_b(\phi_n)}\right)+\alpha > -\left(\frac{k_1(k+k_b(\phi_n)}{k_1+k+k_b(\phi_n)}\right) \tag{22}$$
$$0 < \alpha < G$$

Where $$G=\left(\frac{k_1(k+k_b(\phi_n)}{k_1+k+k_b(\phi_n)}\right)$$

Varying the value of α according to the range in Equation (22) and inserting it into the equation of suspension frequency (20) will give the graph in Figure 2 (a). It can be seen that the lower the value of α, the lower the suspension frequency. The values of k and k_1 that contributes to a low suspension frequency is found by plotting a 3D graph of k and k_1 Vs suspension frequency. Using α=0.00001*G, the 3D plot of k and k_1 Vs suspension frequency for the 2nd mode is shown in Figure 2 (b).



(b)

Figure 2: (a) Effect of α on the suspension frequency of the 2nd mode using k=60 and k1=0 to 60, (b) Graph of k and k1 Vs the suspension frequency of the 2nd mode.

Three cases were studied to see the effect of feedback gain on the frequencies of the beam as follows:

1. No support, no feedback control- beam natural frequencies only
2. With support, no feedback control
3. With support, with feedback control

From Table 3 it can be seen that when there is support, the frequency of the beam at 3, 4 and 5 retained mode is reduced but the 1st and 2nd retained mode has a higher frequency when there is support compared to no support. Meanwhile from Table 4 it can be seen that the frequencies at each retained mode N=1~5 is reduced when there is feedback control.

Table 3: The values of frequency of the beam for modes n= 1~5 with no support and no feedback control as compared to with support but no feedback control

mode	Frequency (No support, no feedback control- beam natural frequencies only)	Frequency (With support but no feedback control)
1	0	4.408
2	3.536	5.437
3	19.11	8.217
4	47.18	14.59
5	87.73	25.34

Table 4: The values of frequency of the beam for modes n= 1~5 with support and with feedback control

mode	Frequency using gd=-653.89 (N=1 retained mode)	Frequency using gd=-661.31 (N=2 retained mode)	Frequency using gd=-761.56 (N=3 retained mode)	Frequency using gd=-899.80 (N=4 retained mode)	Frequency using gd=-952.47 (N=5 retained mode)
1	0.100	0	0	0	0
2	1.0025	0.1	0	0	0
3	5.3511	5.2229	0.1	0	0
4	13.2130	13.1623	12.0953	0.0994	0
5	24.5700	24.5428	23.9871	20.7126	0.0954

Applying active control

The rigid body modes are uncontrollable from the internal force (Preumont, A. & Seto, K., 2002) whereby in this system the internal force is the feedback controller, so only the elastic modes are retained in this system.

The response of linear time-invariant models expressed in the standard state space equation form is

$$\dot{x} = \mathrm{A}x + \mathrm{B}u \tag{23}$$

$$y = \mathrm{C}x + \mathrm{D}u \tag{24}$$

that is, as a set of coupled, first-order differential equations. The solution proceeds in two steps; first the state-variable response x(t) is found by solving the set of first-order state equations, Equation (23), and then the state response is substituted into the algebraic output equation, Equation (24) in order to compute y(t).

Based on the equation of motion of the active suspension unit and the equation of motion of the beam in Equation (5) and (14), the state space representation of the proposed suspension system is:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\dfrac{1}{M+m_b(\phi_n)}\left(k+\dfrac{k_1 g_d}{k_1+g_d}+k_b(\phi_n)\right) & -\dfrac{c}{M+m_b(\phi_n)} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{M+m_b(\phi_n)} \end{bmatrix}}_{B} f_b F_0 \cos \Omega_0 t
$$

$$
y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} f_b F_0 cos \Omega_0 t \tag{25}
$$

Inserting the values of matrices A, B, C and D from Equation (25) into a Simulink block diagram and Matlab m-file, the displacement of the mass M and the beam ($M+m_b(\phi_n)$) for all five retained modes can be observed. Figure 3 shows the excitation force inserted into the system.

The effect of varying the value of negative displacement feedback gain, g_d in the range as shown in Equation (21) can be seen in Figure 4. It is observed that the larger the value of negative feedback gain, g_d the higher the displacement of ($M+m_b(\phi_n)$). Therefore, it is better to use a smaller negative feedback gain, g_d in order to obtain a smaller displacement thus achieving better attenuation of vibration.

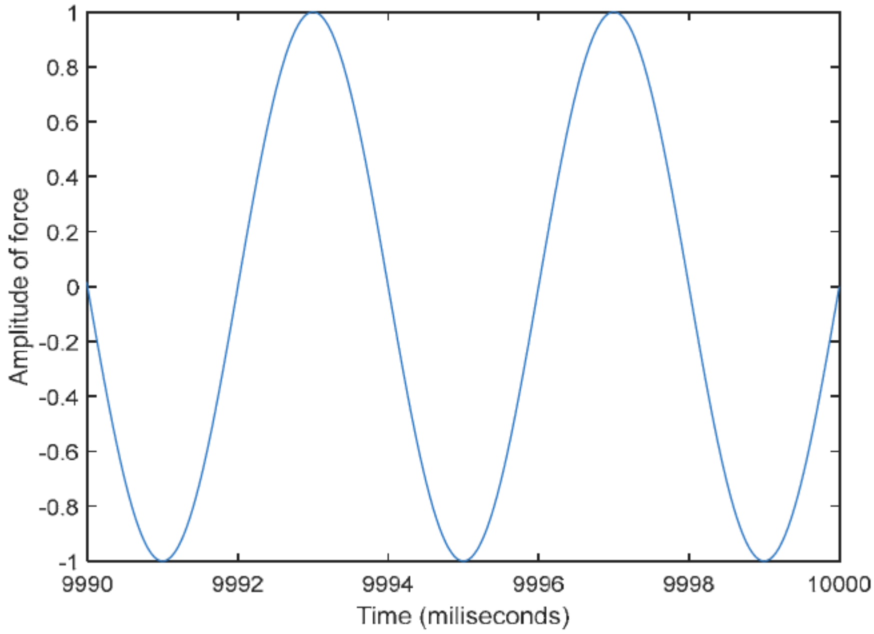


Figure 3: Sinusoidal force input to the system

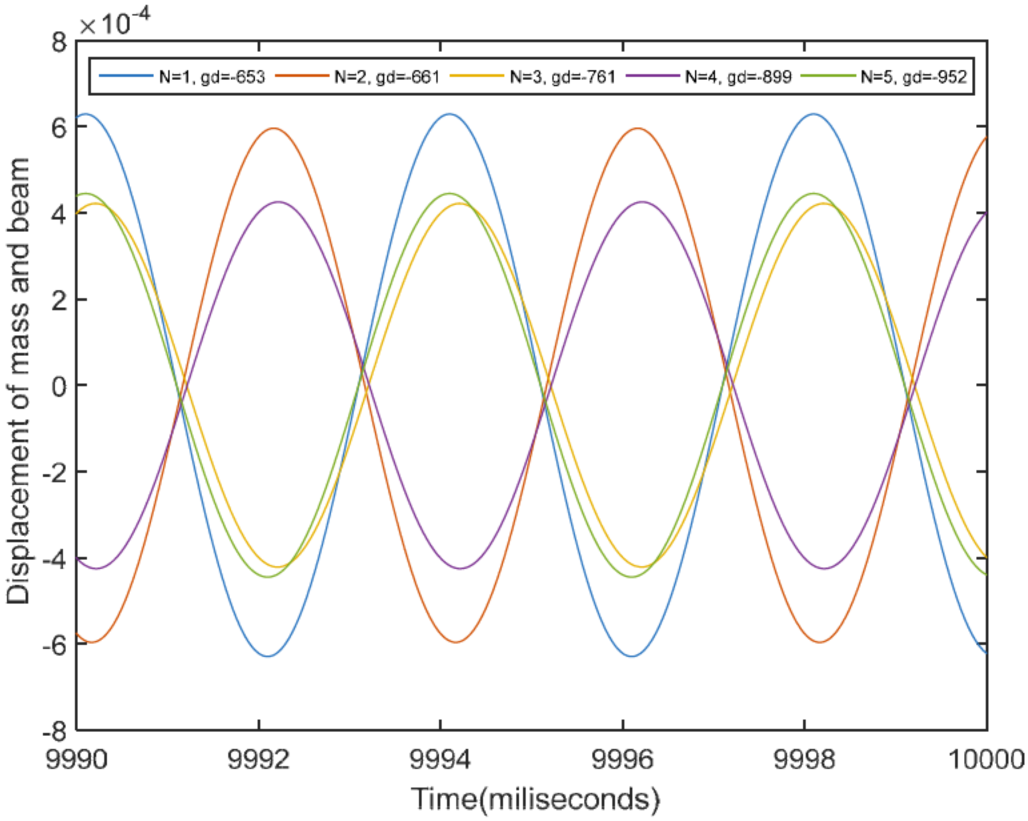


Figure 4: Displacement of mass and beam ($M + m_b(\phi_n)$) for retained modes N= 1~5 using different values of gd.

CONCLUSION

In this work, the interaction between a beam and an active isolator with low suspension frequency is analysed. The beam is shown to provide additional mass, stiffness and force to the active isolator. Suitable values of negative feedback gains that can produce a low suspension frequency were found and the effect of feedback control gain on the frequency of the beam were observed. The dynamic response of the system was studied by varying the value of negative feedback gain. It was observed that the smaller the negative feedback gain, the smaller the displacement of the mass and beam.

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